Clear presentations with words of explanation and all work shown is expected.

Quiz 3 12.5-13.2

Show work clearly with good presentation and words of explanation.
(5 points each)

1) Find and equation of the plane containing points (3,-2,4), (-5,6,0) and (1,5,4)

P Q R

Need point (use any of them) $P(3,-2,4)$

normal vector

$\vec{n} = \vec{PQ} \times \vec{PR} =$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & 8 & -4 \\ -2 & 7 & 0 \end{vmatrix} = \langle 28, 8, -40 \rangle$$

Check! Is this orthogonal to BOTH $\vec{PQ}$ & $\vec{PR}$?

So plane is:

$28(x-3)+8(y+2)-40(z-4)=0$

or any multiple of this

$7x+2y-10z+23=0$

Note: easy to check P, Q, R on plane

P Q

2) Find the point where the line through (-1, -4, 5) and (3,4,-1) intersects the plane x+2y-z+ 1 = 0. Include a screen shot of a computer generated graph of the points, line and plane (rotate to show useful view).

You may use any software you like, but the command on geogebra for point (x,y,z) and direction vector (a,b,c) is
Line[(x,y,z), Vector[(a,b,c)]]

→ see next page.

Line: point $(-1,-4,5)$

$\vec{v} = \vec{PQ} = \langle 4, 8, -4 \rangle$

$\Rightarrow \begin{cases} x = -1+4t \\ y = -4+8t \\ z = 5-6t \end{cases}$

Intersects plane?

$x + 2y - z + 1 = 0$

Substitute line equations into plane

$-1+4t + 2(-4+8t) - (5-6t) + 1 = 0$

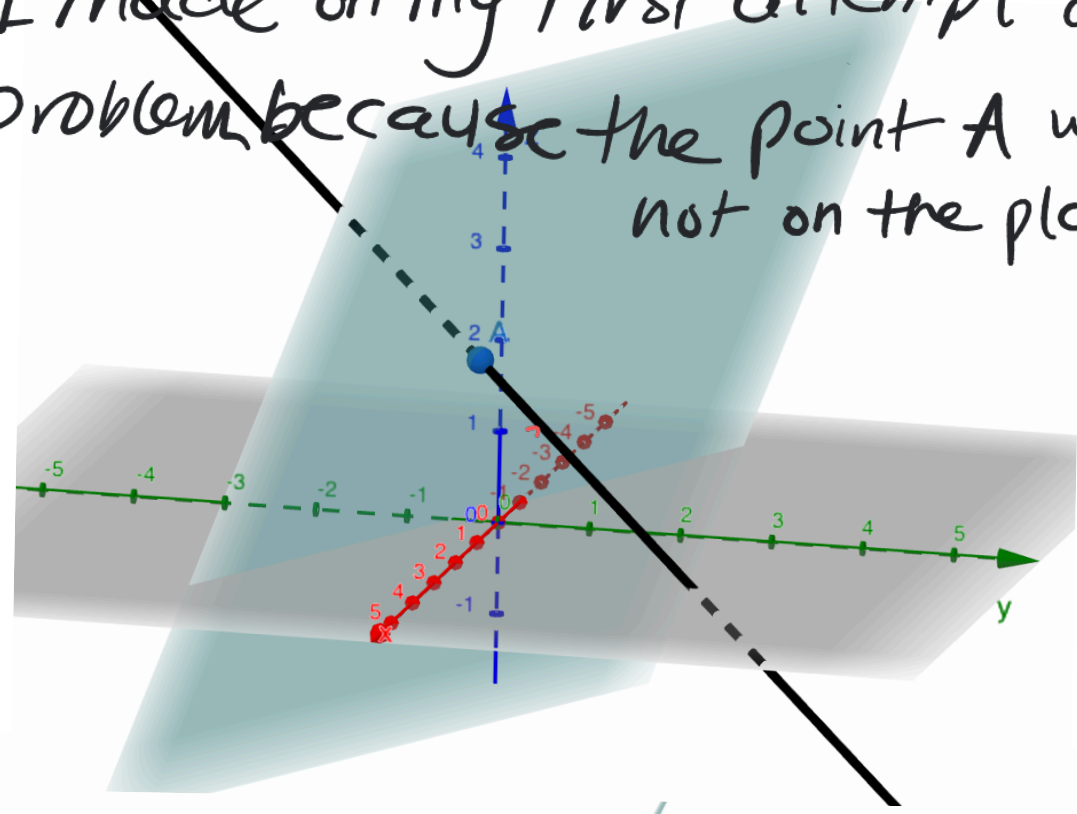
$26t - 13 = 0$

$t = 1/2$ → into line

$(1, 0, 2)$

Graph for #2. The point here is to see whether your answer seems right based on what the graph shows you.

My graph helped me catch a mistake I made on my first attempt at this problem because the point A was not on the plane.



3) Sketch a graph of the following surface in R3.

- Name the surface and give pertinent information such as traces.
  - Use small grids for traces if needed
- Show scale and label axes.

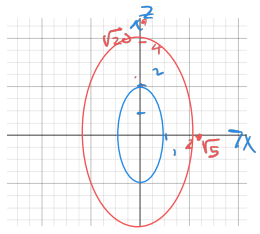
You must show an **accurate** elliptical cross section as discussed in 12.6 video 1 @ 30:40

$$x^2 - y^2 + \frac{z^2}{4} = 1$$

Name of surface: hyperboloid 1 sheet

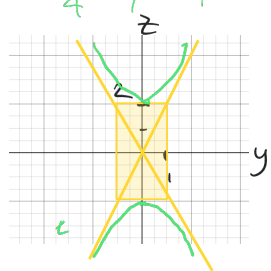
$y=0$

$x^2 + \frac{z^2}{4} = 1$



$x=0$

$\frac{z^2}{4} - y^2 = 1$

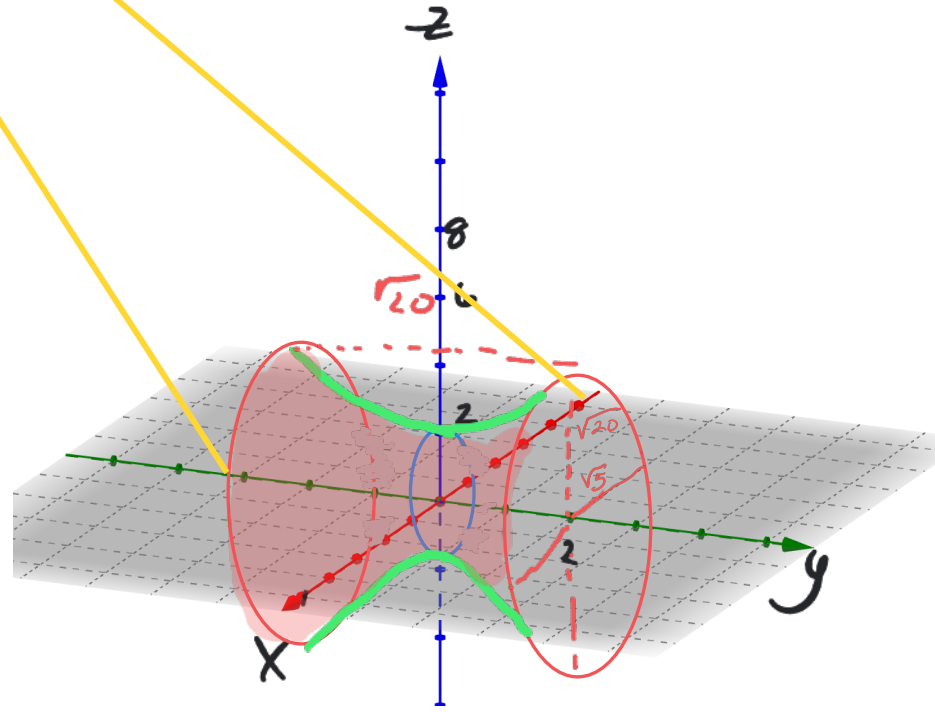


$y=2$

$x^2 - 4 + \frac{z^2}{4} = 1$

$x^2 + \frac{z^2}{4} = 5$

$\frac{x^2}{5} + \frac{z^2}{20} = 1$



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4) Find the equations for the line tangent to the curve $\vec{r}(t) = \langle \cos(2\pi t), t^3, \sqrt{t+7} \rangle$ at the point (1,8,3) (Hint, what is the value of t which corresponds to the given point)

$(1,8,3) = (\cos 2\pi t, t^3, \sqrt{t+7})$

$t^3 = 8$

$t = 2$

$\vec{r}(2) = \langle 1, 8, 3 \rangle$ ✓

$\vec{r}\,'(t) = \langle -2\pi \sin 2\pi t, 3t^2, \frac{1}{2\sqrt{t+7}} \rangle$

Line:

point (1,8,3)

$\vec{v} = \vec{r}\,'(2) = \langle 0, 12, \frac{1}{6} \rangle$

$x = 1$

$y = 8 + 12t$

$z = 3 + \frac{1}{6}t$